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Clarifying Fuzziness: Fuzzy Cognitive Maps, Neural Networks and System Dynamics Models in Participatory Social and Environmental Decision-aiding Processes

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1 Introduction

Addressing global environmental change is a highly complex multi-actor process involving social, economic, political and biophysical dimensions across diverse scales (Vervoort et al., 2012, Cash, 2006, Gibson et al., 2000). There is a need to integrate knowledge across disciplines, sectors and social worlds since every actor has only partial knowledge and expertise about the various aspects of the system relevant to any particular topic (Gibbons et al., 1994, Ulrich, 1993)

Furthermore, different actors have different worldviews, values, visions and aspirations, and beliefs about the actions required to achieve 'positive' outcomes from their perspective (Ulrich, 1987). There is both a moral and a practical need to integrate scientific and expert knowledge with the knowledge and perspectives of lay stakeholders whose lives will be affected by any intervention to inform design of effective policies and actions (Hammond et al., 1999). Involvement of affected stakeholders is an ethical requirement, and in many social and environmental situations, public involvement is mandated by law (Ozesmi and Ozesmi, 2004, Ulrich, 1983). Those stakeholders who will have to implement, maintain and live with any intervention can determine its success or failure according to their level of buy-in (Mikkelsen, 2005). Further, the grounded knowledge they possess, often not captured in scientific and expert knowledge bases, provides insights which support more innovative and appropriate design of policies and actions (Midgley and Richardson, 2007).

From a systems perspective, involving as many diverse scientists, experts and lay stakeholders as possible, is necessary from methodological (who has all relevant knowledge), ethical (who has the legitimacy to decide) and practical (who has the power to implement) standpoints, to ensure the efficiency and effectiveness of any policy or program and minimize unanticipated negative side-effects (Midgley, 2000). This means "new methodological tools are needed transcending the established divide between social and natural sciences, facts and values, objective forecasts and subjective visions" (Kontogianni et al., 2012).

Fuzzy Cognitive Mapping is a technique for modeling complex real world situations, capable of integrating knowledge from multiple actors, disciplines and sectors, including the public (Kosko, 1986, Kosko, 1988, Stach et al., 2010). Fuzzy Cognitive Maps (FCM) can handle both qualitative and quantitative inputs and outputs, and can be used to bridge the divide between qualitative and quantitative analyses of a situation (Kok, 2009, van Vliet et al., 2010). They can be used to represent different mental models of how the world works, and draw various conclusions about belief systems and value systems of different individuals and groups (Gray et al., 2014). This can provide the basis for building shared understanding and clear grounds for negotiation when there is a difference of opinion about what is going on and what should be done to address particular issues in global environmental change (Vaidianu et al., 2014, Wildenberg et al., 2014). As such, they provide a powerful basis for effective stakeholder engagement, providing a common language and representational form for different actors to make sense of complex situations involving multiple perspectives and values, and qualitative and quantitative knowledge bases.

For these reasons FCMs have gained significant popularity across a range of fields of application over the last decade. They have been applied to environmental management, agricultural systems, education, political science, social science, psychology and behavioural science, medicine, engineering, robotics, information technology and telecommunications, business and management (Glykas, 2010, Papageorgiou, 2014). In particular, in recent years there has been an order of magnitude increase in the number of FCM studies and papers published on theory, methodology and applications (Papageorgiou and Salmeron, 2013) including two books devoted to the topic (see Glykas 2010 and Papageorgiou, 2014).

Although the recent popularity of FCM is clear, currently there still exist questions about what constitutes appropriate application of the method, what types of models the FCM methodology produces and how they should be interpreted. For example, within the literature, reference can be found to FCM in terms of three types of mathematical models: fuzzy causal reasoning, neural networks and system dynamics models (Stylios and Groumpos, 1999, Stylios and Groumpos, 2004). In this paper, we try to refine methodological discrepancies in the literature and explain how each of these models work, how they are related and how they are not. In the process we describe some common traps found in literature and practice in the application of these models under the banner of FCM that have significant implications for inferences made for decision-making. Based on insights into these areas, we provide guidelines for the simple, coherent and effective use of FCM and related techniques for social and environmental decision-aiding processes.

The claim of this paper is that internal coherence of the mathematical formulation of FCM activities with conceptual interpretation is important for the quality of the results and the meaning derived from the participatory and analytical exercises that generated the FCMs. Further, we believe disambiguating this information has significant implications for the nature of stakeholder engagement activities and the clarity and simplicity with which they can be facilitated. This paper aims to assist practitioners involved in aiding social and environmental decision-making by providing conceptual and mathematical clarity on the topic of FCMs. Given the proliferation of FCM based studies and programs, the significance of the decisions they are being used to aid, and potential for a degree of perplexity, this is an important and timely contribution.

2 Cognitive Maps and Causal Reasoning

A cognitive map is a visual representation of how a system works from the perspective of those who constructed it. The form of a cognitive map is a directed graph where nodes represent particular concepts and links represent perceived causal relationships between concepts (Axelrod, 1976). These concepts are variable concepts, things that can be caused or not. Kosko gives the example of the model representing social instability rather than society itself, be caused or not, and can cause other things or not, whereas society cannot (Kosko, 1986). The variable concepts can be measurable quantities such as temperature or population, or abstract concepts (more difficult to directly quantify) such as trust, or political will. The values of the variables themselves are not quantified in a cognitive mapping process so it does not matter if they are quantifiable or not.

In participatory processes, cognitive maps are frequently constructed according to the following stages (Ozesmi and Ozesmi, 2004). Firstly, the individual or group constructing the map brainstorms all of the factors that are relevant to a topic of interest. The relevant factors are worded as variable concepts that can be caused and can exert causation, and written on cards. The participant(s) then organize the cards into a logical structure and draw lines between the concepts to indicate where they believe there are causal relationship between the concepts represented. The direction of the arrow indicates that the concept at the base of the arrow, when it exists, causes the concept at the head of the arrow to happen. In this way, participants produce a directed graph, or digraph. An example unsigned graph is shown below.

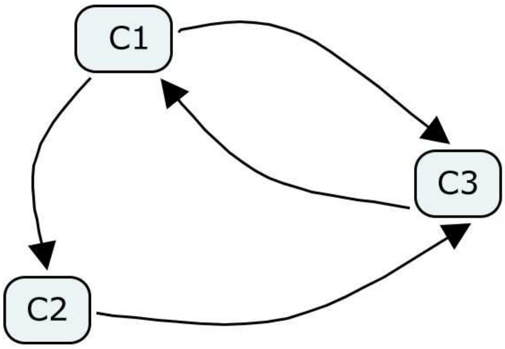

Figure 1 Example Unsigned Cognitive Map

This initial model building stage is extremely useful in participatory processes and is extremely flexible, as participants identify and determine which social, economic, political, physical and ecological aspects of the world are relevant to include in analysis of the issue and which can be left out. The decisions about how to bound the system being modeled, known as “whole system judgments” are subjective and vary by participant(s) (Ulrich, 1983). The way different participants or groups draw the links represent different mental models and belief structures about how the world works and suggest different modes of addressing any given issue (Gray et al. 2014). Given a particular topic, such as deforestation in the Amazon, or management of European water resources, different stakeholders will advocate for the inclusion of different system elements, perceive different relationships between these elements and advocate for different actions, based on their spheres of concern and knowledge. Making these different world views explicit through the selection of elements and linking process, and appropriately negotiating the world view(s) which will be taken forward in further in analysis is key to ethical and effective policy and program development (Ulrich, 1987, Midgley, 2000).

Relationships between concepts, represented by arrows in a cognitive map indicate that a concept causes another concept. The next phase of the participatory process involves participant(s) adding signs to the cognitive map. A “+” is placed on a link between A and B if “A causally increases B” meaning that the participants believe that A happening directly causes B to happen, increases the causation of B, positive causation. A “-” is placed on a link between A and B if “A causally decreases B” meaning that the participants think that A happening directly causes B not to happen, decreases the causation of B, negative causation. If there is no link between A and B, A does not directly causally affect B, meaning that A happening does not directly cause B to happen or exclude it from happening (Kosko, 1986, Kosko, 1988). The variable concepts in this model can be in one of two states, caused or not caused, happening or not happening, often referred to as “active” denoted by 1, or “not active” denoted by 0.

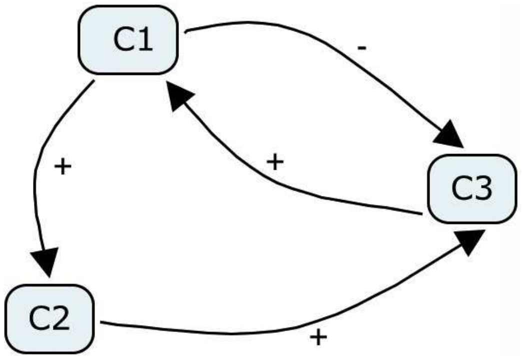

Figure 2 Example Signed Cognitive Map

The term "directly" is important and underlies one of the values of cognitive mapping. In Figure 2 the participants do not believe that C2 directly causes C1, however, by mapping the concepts they discover that they believe that C2 happening causes C1 to happen through a chain of causation. Given specification of certain concepts happening/active we can trace the flow of causal reasoning in the map by tracing the arrows to work out the state of the other variables, that is, whether they are happening (caused) or not happening (not caused). This is the proposed value in cognitive mapping, knowing, or assuming, some initial set of things are happening, causal reasoning provides one answer to what other desirable or undesirable things may happen through the chain of causation, a chain that might not readily be discernible without the mapping.

It is important to clarify that cognitive maps deal with chains of cause rather than chains of effect, though the two are obviously related. In this type of model, the state of the variable is not the value of the variable. This type of model tells you whether something is caused to happen or not (cause), it does not tell you the value of the variable, or how its magnitude is changed, due to changes in the magnitude of other variables (effects).

In a participatory process this is a place where there is potential for different interpretations that require clarification. Cause and effect are often used as proxies for each other. We can think something causes something else because we have observed some effect. The sentences "A causally increases B" and "A causally decreases B" in both Axelrod and Kosko's original presentation of cognitive mapping could be understood by readers to mean "an increase in the value of A causes an increase in the value of B" and "a decrease in the value of A causes a decrease in the value of B" respectively (Axelrod, 1976, Kosko, 1986). This phrasing is in terms of effects on the value of the variable. In fact, in isolation, positive causality is accompanied by positive effect and negative causality is accompanied by negative effect, so this is a reasonable proxy to establish direction of causation. However, it is potentially a dangerous proxy to use as it can pre-frame practitioners and participants to think they are modeling changes in the values of the variables themselves, when in fact cognitive maps model flows of causation. This point ends up being very important for subsequent analysis, since cause and effect have different mathematical models. Cognitive maps, and fuzzy cognitive maps as an extension of cognitive mapping that allows ambiguity in causation, are devices for causal reasoning.

Cognitive maps were not the first devices for mapping causal reasoning. Pathway Analysis in statistics was developed around 1920 (Wright, 1921) and has many graphical and Bayesian progeny, but the simplicity in cognitive mapping makes it attractive for participatory techniques. Cognitive Maps can also be constructed from document analyses as well as in participatory processes. Axelrod and Kosko give numerous examples including a Cognitive Map of Middle East conflict constructed from Henry Kissenger's documents (Kosko, 1986).

However, causation is a difficult concept, it is somewhat of a philosophical minefield (Mackie, 1980, Lewis, 1973, Broadbent, 2012) and in the binary notion of "caused or not caused" we can already observe numerous difficulties in cognitive mapping. In Figure 2 we see that participants believe that C1 happening causes both C3 to happen (through causing C2 to happen) and excludes C3 from happening. Whether C3 happens or not given C1 is happening is ambiguous and our ability to casually reason using this cognitive map is limited. Axelrod (1976) termed the casual relationship between C1 and C3 indeterminate, and one can take the simple algebra he developed no further when indeterminacy is present (Axelrod, 1976).

Axelrod also introduced the use of the adjacency matrix **A** for analysis of cognitive maps. The ij^{th} element a_{ij} of the adjacency matrix **A** gives the value on the link between concept i and concept j, which is either -1, 0 or 1 where -1 represents causal decrease, 0 represents no causal relationship and 1 represents causal increase. The adjacency matrix A for the example cognitive map in Figure 2 is given in Equation 1.

Equation 1 Adjacency Matric for Example Signed Cognitive Map in Figure 2

$$A = \begin{bmatrix} 0 & 1 & -1 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

The usual graph theory measure of centrality of a node in a cognitive map is called the causal conceptual centrality. Various graph theoretic measures and their interpretation for environmental decision making are covered well by Ozesmi and Ozesmi (Ozesmi and Ozesmi, 2004).

3 Fuzzy Cognitive Maps and Causal Reasoning

Kosko explored fuzziness as "an alternative to randomness for describing uncertainty" (Kosko, 1990) (p211). A Fuzzy Cognitive Map (FCM) is a cognitive map modified to capture uncertainty in causal relationships. "Their fuzziness allows hazy degrees of causality between hazy causal objects (concepts)" (Kosko, 1986) (p65).

Where a link exists in a cognitive map, there is no doubt that the causal relationship does always exist, and in the absence of conflicting inputs to a node – indeterminacy - states are certainly “active” or “inactive”. According to Kosko “cognitive maps are too binding for knowledge base building. For in general, causality is fuzzy […] It occurs partially, sometimes, very little, usually, more or less, etc.“ (Kosko, 1986) (p67).

Fuzziness provides a resolution for indeterminacy, for if you were ‘certain’ C1 causes C2, ‘certain’ C2 causes C3, and you were ‘certain’ C1 excludes C3 then something is wrong and only your certainty about these relationships can give way. There is some merit in using fuzzy cognitive maps to interpret “+” and “-“ pathways that lead into a concept as opposing causal forces. With careful interpretation and for the right concepts in some cases “more” caused should win over “less” not caused resulting in, say, the forces aggregating on the side of a caused state. The resolution of indeterminacy enables a form of causal reasoning (Kosko, 1990).

As an example, consider asking your child to clean her room. Asking your child to clean her room does not certainly cause the room to be clean, though it does some of the time, to some extent. After you ask your child to clean her room she may or may not clean it. In the case that she *does* clean it – did you cause that? The answer is “well, kind of… to a degree”. Perhaps you think that if you didn’t ask her she probably wouldn’t have done it at all so in some way you contributed to it being clean by asking, though you are in no way fully caused it. The child is not a robot, no matter what constraints you put on a child to behave in particular ways you cannot fully cause their behavior.

Now introduce her favorite television program. In the absence of your asking the television would almost always prevent the room being cleaned. Usually, in the absence of the television, your asking would mostly cause the room to be cleaned, but in competition with the television causing the room not be cleaned, overall your asking loses out against the television. The room, though it could end up being cleaned or not cleaned, leans toward the not cleaned state. In the original sense of fuzzy sets we would say that the state of the room (which is uncertain/ambiguous because your asking does not always cause it to be cleaned, and there is a competition of causal forces) belongs more to the cleaned state than the not cleaned state. If your asking and television is of equal causal influence, then the state of the room should remain the most ambiguous and belong equally to the cleaned and not cleaned state.

Fuzzy cognitive maps are fuzzy digraphs, an involved mathematical concept, but we will describe their simplest mathematical form and how they can be used for causal reasoning in participatory processes.

In FCMs a link is drawn where a causal link potentially exists, to some degree. A weight between 0 and 1 is placed on the link to capture the degree of uncertainty in the causal relationship. FCMs show haziness in causality rather than haziness in magnitude of effect. The stronger the potential that causation will occur, the closer the weight is to 1 and the weaker the potential that causation will occur the closer the weight is to 0. If you know your child will certainly clean her room when asked you would put a 1 on the link between “asking” and “clean room”, if you know your child certainly does not listen to you at all, and there is no link between you asking and the room being cleaned, you would not put a link, which is equivalent to 0, otherwise you may put something in (0,1). The sign of the causality is established according to the convention described in the previous section, thus ultimately producing a number on the link in [-1,1]. An example FCM is shown in Figure 3.

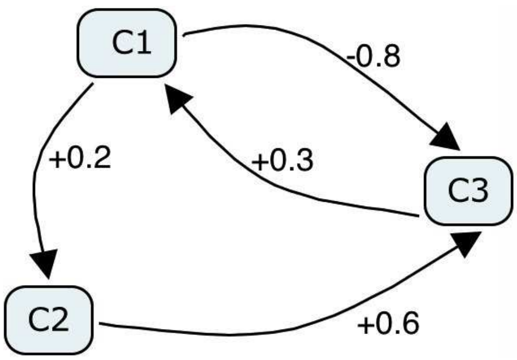

Figure 3 Example Fuzzy Cognitive Map

In the example in Figure 3, potentially C1 could be fishing, C2 decline in fish stocks, C3 poverty. FCMs permit feedbacks because it is valid to be uncertain whether one concept causes another or is caused by another, or the direction of causality is ambiguous because the model has not specified time evolution and so it is possible for concepts to cause each other under different conditions and time that the model does not resolve. Ideally, of course, you would include this temporal resolution in more refined conceptual modeling, but it is possible for an FCM to represent the ambiguity. There is no trick or magical gain in this, all that will happen by using an FCM is that you’ll get a more ambiguous answer to your analysis, requiring more cautious interpretation, which is potentially more honest for complex real-world decision-aiding.

Stakeholders may tend to estimate the strength of causality by the number of times something happens in response to something else in their experience, that is, using effect as a proxy for causality (Wildenberg et al., 2014). This is alright but potentially dangerous as it leads to the potential interpretation of the link as the conditional probability of an event given another and while there is a relationship between fuzziness and probability, and they are both ways of dealing with uncertainty, they are not exactly the same thing and the aim is not to estimate probabilities (Kosko, 1990).

The adjacency matrix **A** of the FCM is defined in the same way as for cognitive maps, so that the element in row *i* column *j*, a_{ij}, is equal to the value of the number on the link between concepts *i* and *j* and is in [-1,1]. Thus, the adjacency matrix for the FCM in Figure 3 is given in Equation 2.

Equation 2 Adjacency Matrix of Example Fuzzy Cognitive Map in Figure 3

$$A = \begin{bmatrix} 0 & 0.2 & -0.8 \\ 0 & 0 & 0.6 \\ 0.3 & 0 & 0 \end{bmatrix}$$

The states of the concepts in the cognitive map are also made fuzzy. In cognitive maps the variables can only have value 0 or 1 meaning caused (active) or not caused (inactive). In FCMs the state can be caused, not caused, or somewhere hazy in between (we are not sure if it is caused or not), or in numerical terms, anywhere in [0,1]. The state is specified as a row vector **S**, where S_i is the state of concept *i*. The states of the concepts is "fully active": 1, "active to some degree": in (0,1), or "not active": 0; *not the value of the variable itself*. To continue the previous example, we don't know that the room is partially clean and how clean it is (which would be a description of effect), but we are uncertain whether it has been cleaned or not been cleaned.

The FCM can be used to make inferences about causation from any given set of initial conditions. These initial conditions may represent a policy or program intervention that activates particular concepts. Making inferences about what is caused to some degree, and what is not caused by the intervention (according to a particular worldview represented by the whole FCM) is based on calculation of how this activation subsequently spreads through the graph.

Let the total direct causal input to node *i* be T_i. The total direct causal input to node *i* at any point during the activation spreading process, is the sum of the *active* causalities to the node. The active causalities are obtained by multiplying the activation state of each preceding node by the weight that specifies the degree of causal influence it exerts when active. A node that is not active will not exert a causal influence on nodes it is subsequently connected to. Negative and positive causalities act to cancel each other out in this summation.

Equation 3 Total Causal Input to a Node

$$T_i = \sum_{k=1}^{N} S_k a_{k,i}$$

The most T_i could ever be is N if all N concepts are fully active and exert full positive causality on node *i*, and $-N$ if all N concepts are fully active and exert full negative causality on node *i*. If T_i is 0 then all direct causal forces on the node *i* are equal, or the node *i* has no causal connection at all.

The level of direct causal input needs to be transformed into knowledge about whether the concept at *i* is a degree of "activated" or "not activated" as a result. Accordingly, the value of the state is in [0,1], though in general the input to the node T_i could be in [-*N, N*] a squeezing function is used. In the simple case where the state is not fuzzy and must be either 1 or 0 that is "activated" or "not activated", threshold functions are used. In the case where a concept can be partially active, continuous squeezing functions are used to transform the sum of the inputs to a value between 0 and 1. This squeezing function is known as the activation function since it is used to calculation the activation level of node *i*.

The most common activation function used in FCM is the unipolar sigmoid function given by Equation 4, which squashes its content within [0,1]. The slope of the unipolar sigmoid function is defined by the choice of the parameter lambda shown below. Its value must be established by the FCM designer. For high values of lambda parameter, the sigmoid approximates a discrete function (threshold function), and for lower values of lambda, the sigmoid approximates a linear function, while values of lambda closer to 5 generate a balanced degree of fuzzification of the state (Bueno and Salmeron, 2009). Other squeeze functions used include the hyperbolic tangent.

Equation 4 Equation for Unipolar Sigmoid Function

$$f(x) = \frac{1}{1 + e^{-\lambda . x}}$$

The choice of the form and parameterization of squeezing function has been shown to have significant impact on results, sometimes reversing certain conclusions (Papageorgiou and Salmeron, 2013, Knight et al., 2014). This relies on the judgment of the modeler and feedback with participants to adjust and is a demonstration that robust analytic techniques should be used. Note the important property of most continuous squeezing functions, especially the sigmoid function, that *f*(0) = 0.5. So if all causal forces on a node cancel out, we get the correct interpretation that the node state caused by these forces is in its greatest degree of ambiguity, that is, belonging equally to the state of "activated" and "not activated". This is the fuzzy logic equivalent of Axelrod's original indeterminacy. It is also the correct interpretation if the node has no causal connections at all, because our knowledge about the activation or deactivation of other disconnecting concepts imparts no knowledge about the state of the node in question, so its state due to causation from the activated concepts could be anything.

Kosko calculates each subsequent value of the state S_i of concept *i* of the concepts given an initial input by

Equation 5 (Kosko, 1988, Stylios and Groumpos, 1999):

Equation 5 Activation Propagation Through an FCM

$$S_i(j+1) = f\left(\sum_{k=1}^{N} S_k(j).a_{k,i}\right)$$

Where j is the index denoting iteration number. For the entire state vector $\boldsymbol{S}$ this is equivalent to the matrix equation given in Equation 6:

Equation 6 Matrix Equation for Activation Propagation Through an FCM

$$\boldsymbol{S}(j+1) = f(\boldsymbol{S}(j).\boldsymbol{A})$$

An initial activation state $\boldsymbol{S}(0)$ will directly cause $\boldsymbol{S}(1)$, which will subsequently cause $\boldsymbol{S}(2)$ and so forth. To follow the chain of causation through the FCM we have to repeatedly apply Equation 6.

This calculation rule, the original introduced by Kosko, calculates the state of each concept based only on the influence of interconnected concepts at each moment. This is equivalent to a memoryless process. Various authors have suggested introducing memory into the calculation, so that subsequent states depend not only on the sum of active causalities, but also on any (weighted) number of past states. The following equation is the most common and includes the preceding state in the calculation of each subsequent state with weight 1 (Papageorgiou and Salmeron, 2013, Papageorgiou and Salmeron, 2014, Stylios and Groumpos, 2004, Stylios and Groumpos, 1999):

Equation 7 Activation Propagation Through an FCM with One State Memory

$$S_i(j+1) = f\left(S_i(j) + \sum_{k=1}^{N} S_k(j)a_{k,i}\right)$$

For the entire state vector $\boldsymbol{S}$, the matrix version of this equation is given in Equation 8:

Equation 8 Matrix Equation for Activation Propagation Through an FCM with One State Memory

$$\boldsymbol{S}(j+1) = f(\boldsymbol{S}(j) + \boldsymbol{S}(j).\boldsymbol{A}) = f(\boldsymbol{S}(j)(\boldsymbol{I} + \boldsymbol{A}))$$

Thus, including the previous state in the calculation of the subsequent state (according to Equation 7) can be achieved by putting 1s along the entire diagonal of the adjacency matrix. If weights are used to temper the effect of the previous states, the weight on preceding state i, k_i can be set as a_{ii} (Stylios and Groumpos, 2004).

After repeated calculation of each successive state of the FCM with Equation 6 or Equation 8, most FCMs will settle on either a single stable activation state or a stable recurrent pattern of activated concepts, known as a stable limit cycle. "In practice it will converge after very few iterations" (Kosko, 1988) (p381). This is the inference the FCM provides about causation for a given input. It gives us an uncertain understanding of what will be caused in certain conditions given uncertain causal relationships.

Combining Knowledge Bases in FCM

FCMs can be used to calculate how different worldviews suggest different inferences about the world. Maps produced by different experts or groups can also be combined to produce a larger representative knowledge base of all of the participants. "Larger expert sample sizes should produce more reliable knowledge bases… The strong law of large numbers ensures that as expert sample size increases, knowledge base reliability increases" (Kosko, 1988). Note that this does not mean the individual maps will produce the same result as a combined map, each represents different world views and are likely to give different results, it means that as the number of experts involved increases, the form of the overall FCM in terms of elements and weights will eventually stabilize and constitute a good representation of the state of extant knowledge on a topic.

Kosko suggests that each expert/group can have a credibility weight w_i in [0,1] representing the degree of certainty we have a prori about their knowledge, which combines well with the overall graph since an FCM itself represents uncertain knowledge and thus we can combine uncertain knowledge sources (Kosko, 1988). Combined weighted FCMs can reflect the different levels of expertise on the topic. Of course the quantification of credibility itself reflects particular worldviews about legitimate sources of expertise and knowledge. In any case, equal weights of 1 could be given where appropriate.

According to Kosko, the method for combining maps is as follows: include all elements of all maps in a larger augmented matrix. The ij^{th} element of the augmented matrix A is the weighted average of all of the entries for that position (0 in the case where the concepts/link did not exist in a particular map) and "the Kolmogorov strong law of large numbers ensures that, with probability one, as sample size increases, A approaches the underlying matrix distribution of means" (Kosko, 1988). Combining connection matrices is a type of adaptive learning, adjusting the entire FCM as new information comes to light. Changing the elements, links and weights in an FCM is what is referred to when Kosko talks about the dynamics of the FCM. The dynamics of the FCM are not the same as the dynamics of the underlying variables, this will be addressed more in Sections 4 and 5.

In summary, FCMs are a practical way to represent uncertain causal knowledge of different actors individually, or in groups, and the maps produced can be examined in isolation or combined. They can give us a lot of information about differences in how stakeholder knowledge is structured and what this suggests for policy and action (Gray et al., 2014, Ozesmi and Ozesmi, 2004). The results of FCM calculations (the FCM inference) must be used carefully since, as discussed above, these results are sensitive to choice and parameterization of squashing function, the amount of memory the state of the FCM has, how different maps are combined and without proper sensitivity analysis testing the robustness of conclusions there is a risk of using gibberish results or of tweaking the map to produce what you already think or want.

4 Fuzzy Cognitive Maps and Artificial Neural Networks

FCMs as described above, are structurally analogous to Artificial Neural Networks (ANNs) (Kosko, 1988, Papageorgiou and Salmeron, 2013, Stylios and Groumpos, 1999). Both FCMs and ANNs are weighted directed graphs where the nodes are activated or not as a function of summation of the input to the node, known as the activation function. The activation function most commonly takes the form of a threshold function or unipolar sigmoidal function in both FCMs and ANNs.

The structure of ANNs is designed to mimic the physiology of the human brain: nodes represent neurons, which can have any number of inbound connections known as dendrites, each neuron has only one outbound connect known as an axon, which can subsequently branch. Thus nodes can have any number of inbound and outbound connections. These connections act to propagate electrochemical stimulation between neurons. If the sum of the inputs to a node/neuron is greater than a threshold the neuron becomes “active” and “fires” an electrochemical potential thus transmitting a signal to subsequent nodes. If the neuron is not “active” it does not “fire” and no signal is transmitted to subsequently connected neurons. The strength of the signal transmitted to the next node is mediated by the weight on the link; in an analogous way to the way certain neurochemicals such as dopamine and serotonin inhibit or excite synaptic connections.

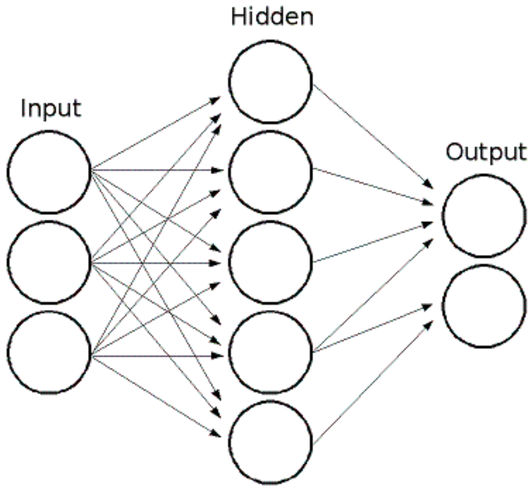

Figure 4 Standard hidden layer neural network structure (OpenCV API Reference, 2014)

In most neural networks, only the input and output nodes have specific interpretation, the rest of the nodes - often called hidden layers - have no specific interpretation as concepts; they simply act as part of a distributed processor, whereas in FCM all the nodes have a meaningful interpretation.

ANNs are numerically “trained” for specific problem contexts. An unsupervised learning algorithm, such as the back propagation algorithm, is used to adjust the weights on the links in the ANN and the parameters of the threshold functions on each node so that historical input data matches historical output data. The issue of when to stop training ANNs has received considerable attention in literature and practice. The ANN can then be used to predict outputs the outputs of potential future input sets. ANNs represent a data driven way of modeling real world situations in cases where mathematical equations describing the system are not feasible to construct and solve.

In many social and environmental decision making processes FCMs developed by stakeholders, while used to calculate the causal flows resulting from different inputs, themselves remain static in terms of elements, links and weights. Without data driven unsupervised training the analogy between FCMs and ANNs ends at structural resemblance, as unsupervised learning is an important feature of ANNs.

A number of data driven techniques for unsupervised learning of FCMs have been developed such as the Hebbian Learning Algorithm (Kosko, 1988), Active Hebbian Learning Algorithm (Papageorgiou et al., 2004) and Song et al.’s fuzzy neural network method (Song et al., 2010). In this case FCMs resemble neural networks more closely. Some authors see data driven training algorithms for developing the link values in an FCM see these methods as a method for overcoming the “subjectivity of stakeholder generated maps. For other authors capturing different worldviews and their implications may be the very point of using FCM and therefore data driven training which overrides the knowledge about world view contained in the FCMs of various groups would not be desirable. Modelers responsible for large global climate, economic and environmental models can simply draw the FCM representing the elements and connections assumed in their model to make their modeling assumptions transparent to stakeholders and comparable to their own FCMs.

These data driven FCM training techniques have rarely been applied in stakeholder driven social-environmental decision processes, though this is certainly possible and potentially a way of incorporating long term ecological research data sets into the analysis together with stakeholder knowledge (Wildenberg et al., 2014). Thus, most of the time, the FCMs used are static rather than dynamic. These FCMs are only structurally analogous to neural networks and do not undergo learning.

Reference to the “dynamics of the FCM” refers to the dynamics of the links and weights during a training process rather than the dynamics of the underlying system (Kosko, 1988). This is different to systems dynamics models which model stocks and flows – FCMs do not model stocks and flows, they do not tell us the values of the variables themselves, or how much they are changing (effect) they tell us about what is caused or not in certain conditions (cause). Interpreting the state of the node as the value of the concept itself leads to the development of system dynamics models, which are not the same as fuzzy cognitive maps, not necessarily even fuzzy and do not have the same relationship with neural networks. Nevertheless, in some circumstances it is desirable to develop simple systems dynamics models and it is important to clarify their relationship with FCMs, and how they are effectively operationalized in participatory social and environmental decision-making processes. This is described in Section 5.

5 Fuzzy Cognitive Maps and System Dynamics

An alternative interpretation of a weighted graph such as the one in Figure 3, is as a system dynamics model, tracking stocks and flows, changes in the real values of variables as a result in changes in the values of other variables, rather than changes (Kok, 2009, Knight et al., 2014, van Vliet et al., 2010, Stylios and Groumpos, 2004). This gives a different physical meaning to the weights on the links and the values of the nodes, and importantly to the mathematical method for calculating model outputs and physical interpretation of results than in CM and FCM. In this interpretation the weight on the link is interpreted as a measure of *how much* effect one concept has on another rather than *how certain* participants are that one concept will cause another (Carvalho and Tome, 2001). That is a_{ij} represents the magnitude of the effect that a change in the real value of concept C_i has on the real value of concept C_j. In this interpretation, stakeholders are asked, for example, if C_i has a low, medium or high impact on C_j, rather than how certain they are that there is a positive or negative causal relationship between C_i and $C_{j,}$ (Kok, 2009, Wildenberg et al., 2014, Vaidianu et al., 2014, Gray et al., 2013). That is, a_{ij} a measure of "how much effect" rather than "how much uncertainty" about cause.

The state of the system is interpreted as the real values of the variables themselves rather than the activation level. These real values could be normalized between 0 and 1 or -1 and 1 (assuming finite range of the variables). However, the reasons for doing so are different to in FCM where the reason for the transform is because of the interpretation of state as on/off, activated/not activated, or something in between.

The initial state in this case is thus a quantification of the initial values of the variables in real terms rather than an estimate of how certain we are whether they are "happening or not" in the first place. This potentially makes implementing this approach harder in the case where values of variable are harder to estimate, though as will be seen, an estimate of initial value is not always needed if what is wanted is just to explore the change produced by certain policies or programmatic interventions. Again there is nothing fuzzy about the interpretation of state in this case; it is simply a single real value of the variables, normalized in some way.

To understand how this system dynamics interpretation of the graph works and relates to FCMs and their associated calculations, consider the graph fragment shown in Figure 5. This fragment can be read as a change in the value of C_1 of magnitude ΔC_1 causes a change in the value C_2 of magnitude $\Delta C_2 = \Delta C_1.a_{12}$. This is summarized in **Fout! Verwijzingsbron niet gevonden.** with index j to denote the sequential nature of each change, i.e. first the change in C_1 occurs, then this causes a change in C_2. Sequence versus time in this interpretation of the graph will be discussed shortly.

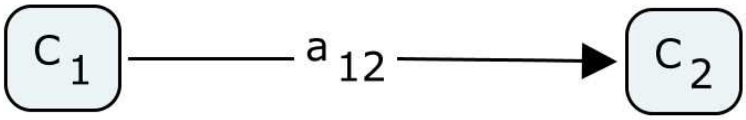

Figure 5 Example Graph Fragment

Equation 9 Equation Equivalent to Graph Fragment Shown in Figure 4,

Change in C2 as a result of Change in C1

$$\Delta C_2(j+1) = \Delta C_1(j)a_{12}$$

The total change in C_2 will be the sum of all of the change inputs to C_2 from connections with other variable concepts C_1 … C_N, where *N* is the total number of concepts, as shown in Figure 6 and Equation 10.

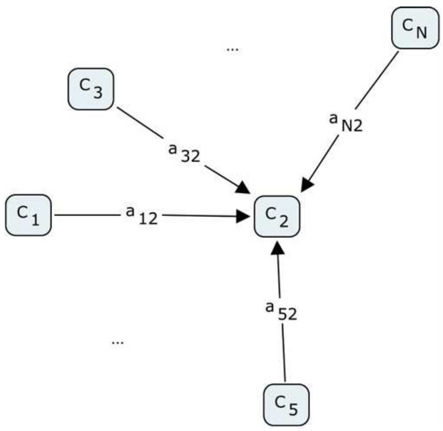

Figure 6 Example Graph Fragment Illustrating Inbound Connections to Node 2

Equation 10 Equation for Total Change to Concept 2 at iteration j

$$\Delta C_2(j+1) = \sum_{i=1}^{N} \Delta C_i(j).a_{i2}$$

Similarly, we can generate a set of equations for the change in value of each of the variable concepts:

$$\begin{aligned} \Delta C_1(j+1) &= \sum_{i=1}^{N} \Delta C_i(j) a_{i1} \\ \vdots \quad &= \quad \vdots \\ \Delta C_N(j+1) &= \sum_{i=1}^{N} \Delta C_i(j) a_{iN} \end{aligned}$$

This is equivalent to the matrix multiplication given in Equation 11.

Equation 11 Matrix Equation for Linear System Dynamics

$$\boldsymbol{\Delta C}(j+1) = \boldsymbol{\Delta C}(j) \cdot \boldsymbol{A} = \boldsymbol{\Delta C}(0) \cdot \boldsymbol{A}^{j+1}$$

Thus, Δ***C***(0) is the initial perturbation applied to the system. Under the systems dynamics interpretation of the weighted directed graph generated by stakeholders, we simply have a static map of how things are connected, the system is not changing unless it is pushed. This is why if Δ***C***(0) = **0**, all subsequent iterations of Δ***C***.***A*** are also **0**. This doesn't mean the state is 0 it means the change is 0. It operates like a wind-chime.

The first time the matrix Δ***C***(0) is multiplied by ***A*** it we obtain the direct impacts of the change Δ***C***(0) on each of the variables: Δ***C***(1) = Δ***C***(0) · ***A***, these changes themselves subsequently cause other changes in each of the variables: Δ***C***(2) = Δ***C***(1) · ***A*** = Δ***C***(0) · $\boldsymbol{A}^2$; and so forth such that j^{th} order effect is given by Δ***C***(j) = Δ***C***(j-1) · ***A*** = Δ***C***(0) · $\boldsymbol{A}^j$. Using Equation 11 it is possible to calculate and plot the successive order effects Δ***C***(j) for j = 1,…,∞ which result from an initial perturbation Δ***C***(0) without knowing the initial state of the system ***C***(0) (values of the variables themselves in this case). The cumulative change which has occurred by iteration *j* is obtained by summing each successive orders of effects, including the initial perturbation itself, which itself represents a change in the initial values of the variables. This can be calculated without knowing the initial state of the system ***C***, and is given in Equation 12.

Equation 12 Cumulative Change In Variable Values at j^{th} Iteration

$$\boldsymbol{\Delta C}_{cum}(j) = \boldsymbol{\Delta C}_{cum}(j-1) + \boldsymbol{\Delta C}(j) = \sum_{i=0}^{j} \boldsymbol{\Delta C}(i) = \sum_{i=0}^{j} \boldsymbol{\Delta C}(0) \cdot \boldsymbol{A}^i$$

The total cumulative change that occurs as a result of an initial perturbation is calculated by summing together *all* successive order effects including the initial change itself, as given in Equation 13. Again, knowing the total change that results from an initial perturbation depends only on knowing the perturbation, not on knowing the initial state, or any subsequent states.

Equation 13 Total Change in Variable Values Resulting from Initial Change ∆C(0)

$$\boldsymbol{\Delta C}_{TOTAL} = \sum_{i=0}^{\infty} \boldsymbol{\Delta \mathrm{C}}(\mathrm{i}) = \sum_{i=0}^{\infty} \boldsymbol{\Delta C}(0) \cdot \boldsymbol{A}^{i}$$

If the initial state vector ***C***(0) - as opposed to the initial change vector ∆***C***(0) - is known, that is, the initial values of the variables themselves, each successive state value can easily be calculated according to Equation 14. If the initial state vector ***C***(0) is not known, it is still possible to calculate the change $\Delta \boldsymbol{C}_{TOTAL}$ caused by any disturbance applied as an initial change vector ∆***C***(0).

Equation 14 Equation for Variable Values

$$\boldsymbol{C}(j+1) = \boldsymbol{C}(j) + \Delta\boldsymbol{C}(j)$$

Since ∆***C*** is a row vector and ***A*** is the adjacency matrix of the graph, the similarity between Equation 11 above and the Equation 6 for propagation of activation through an FCM can easily lead to the two equations being mistaken for each other, especially if the above interpretation (un-fuzzy systems dynamics) is being thought of as an FCM, though the two graphs and equations have very different meanings.

Various participatory social and environmental decision-aiding processes conducted as FCM exercises have used the systems dynamics interpretation described here, and accordingly asked participants to estimate the strength of effect rather than the degree on uncertainty of each link (Kok, 2009, van Vliet et al., 2010, Vaidianu et al., 2014, Wildenberg et al., 2014, Stylios and Groumpos, 2004) Some authors are vague mentioning “strength of relationship” which could be interpreted either way by participants so we cannot be sure (Kontogianni et al., 2012, Ozesmi and Ozesmi, 2004, Gray et al., 2013). Some authors then continue to use the mathematical formulation of FCMs interpreting the row vector multiplying the adjacency matrix as some form of state, either activation level (Ozesmi and Ozesmi, 2004, Kontogianni et al., 2012), or the values of the variables themselves (Kok, 2009, van Vliet et al., 2010), and some continue to apply the squeezing function to the inputs to each node (Wildenberg et al., 2014, Vaidianu et al., 2014).

In the systems dynamics interpretation we are dealing with magnitudes of change that results from a perturbation, it is important to know if this produces unstable effects. Runaway change would be something we would be concerned about and therefore reducing and bounding the change possible at each node is not desirable. For example, if increasing fertilizer subsidies would produce runaway deforestation it is important to know this rather than have the change bounded by a squeezing function. One type of bounding of change that does make sense: for variables of finite range, the variable cannot change to less than its minimum value or higher than its maximum value and this places some limitation on the total amount of change possible – once the forest is completely gone, the decrease in forest must drop to zero. This is most accurately captured by using functions with bounded support. Note that for any variables without clear bounds on their values, such as the average temperature of a planet, there is no reason the value of the variable must stay between 0 and 1 or -1 and 1 or that the total change in their value at any iteration must be between -1 and 1.

Introducing the squeezing function does introduce non-linearity into the system, but not in a practically meaningful way. When it is desirable to include non-linear relationships between variables in a system dynamics model, it makes sense for the non-linearity to be representative of the real system being modeled. A simple way to do this would be to ask participants to draw the shape of the relationship between any two variables on each link. In some cases the shape may be sigmoidal, in others exponential, power law, linear and so forth. An image of this is shown in Figure. This is actually an important step in developing better systems dynamics models since the assumption of linear relationships between the variables is non plausible to most participants.

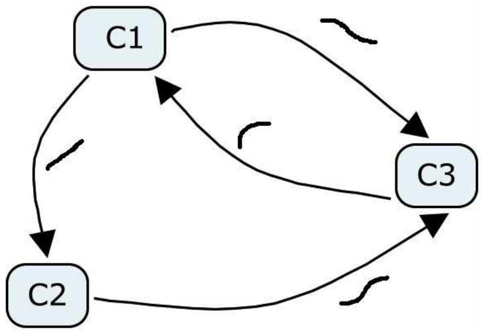

Figure 7 Example graph with non-linear relationships between variables

The system dynamics interpretation of the graph described in this section is not a dynamic model in terms of time, it represents inter-variable dynamics rather than time dynamics. The graph represents how changes in certain variables produce changes in others, but without further information the time frame these changes take place in is not directly represented. Although time is not explicitly part of this formulation, sequences of events are. When a perturbation $\Delta\boldsymbol{C}(0)$ is applied to the system, the first order effects are those changes in variable values which occur as a direct result of the initial perturbation, given by $\Delta\boldsymbol{C}(1) = \Delta\boldsymbol{C}(0) \cdot A$. These first order changes then represent a perturbation which itself has effects, those effects are the second order effects of the initial perturbation $\Delta\boldsymbol{C}(2) = \Delta\boldsymbol{C}(1) \cdot A = \Delta\boldsymbol{C}(0) \cdot A^2$. The third order changes are those that occur as a result of the second order effects, these are given by $\Delta\boldsymbol{C}(3) = \Delta\boldsymbol{C}(2) \cdot A = \Delta\boldsymbol{C}(1) \cdot A^2 = \Delta\boldsymbol{C}(0) \cdot A^3$ and so on. We have no concept of the time frame that these sequential changes take place in, seconds or millennia. However, the concept of sequence is still relevant even without time. We are examining sequences of effects of variables on each other within a system.

Although time is not explicit it plays an important role in the estimation of the strengths of relationships between elements. Notice that the strength we would assign to the impact of climate change on crop yield is less then fertiliser availability in the short term but higher in the longer term since evidence shows that continuously increasing fertiliser application does not result in continuously increasing yields. People necessarily have an implicit time frame in mind when assigning relative values to the links on the FCM. Unless the timeframe for comparison of effects is specified in the participatory process, participants may disagree simply because they are imagining relative effects over different time frames. Ultimately each iteration matrix multiplication represents some "time unit", it would be helpful if this were specified in the participatory process.

Potentially due to the comparison with FCM equations, many authors refer to the row vector $\Delta\boldsymbol{C}$ in Equation 11 as the state vector $\boldsymbol{C}$ (Kok, 2009). This leads to post multiplying the initial state of the system $\boldsymbol{C}$ by the adjacency matrix, which is equivalent to subjecting the system to a disturbance whose magnitude is equal to the state of the system itself! This also leads to the successive changes in Equation 11 being interpreted as successive states of the system, this is not mathematically correct in the system dynamics interpretation of the graph. Accordingly, various authors have calculated and plotted successive change vectors $\Delta\boldsymbol{C}(j)$ for j = 0,1,2,… and interpreted this plot as a plot of the state of the system, that is the values of the variables $\boldsymbol{C}$ at each successive iteration (Kok, 2009, van Vliet et al., 2010, Knight et al., 2014). Thus, if the perturbation eventually dies out, this causes anxiety since researchers and participants alike know that the value of all state variables themselves cannot all drop to 0! However this is not the case as this graph does not represent successive values of the variables themselves but successive changes in the values of the variables.

This has further led to some researchers calibrating the values of the adjacency matrix to produce constant non-zero values for $\Delta\boldsymbol{C}$, which is being interpreted as $\boldsymbol{C}$. This is the same as producing on-going displacement, that is, an unstable system. This is often done by placing 1's along the diagonal of the adjacency matrix, in positions corresponding to variables that are thought of as "drivers", in order to produce an on-going displacement (Kok, 2009, van Vliet et al., 2010, Knight et al., 2014). Note that in this system dynamics formulation, a_{ii} is the magnitude of the effect that a change in C_i has on itself. Setting a_{ii} = 1 is equivalent to saying concept C_i has a 100% positive forcing effect on itself. In practice a change in the value of the variable could cause a further change in the value of the variable, but there is no reason it should always be positive and always be 100%. Using separate terms for state vector and change vector could avoid these potential issues.

More generally, since we are dealing with magnitude of effect in this interpretation, there is in fact no reason the number on the link should be limited within [-1,1]. In FCM the reason the weights are within [-1,1] is because +/-1 implies certainty that the causal relationship exists and 0 implies certainty that a causal relationship does not exist and anything in between represents a degree of uncertainty about causality. In a system dynamics model, a small change in one variable could potentially produce a much larger change in another variable, leading to weights potentially anywhere in [-∞, ∞].

Whether or not the successively higher order effects of the initial perturbation die out (that is go to zero), stabilise at finite values, or tend to infinity depends on the behaviour of A^n as n tends to ∞. This depends on the spectral radius of the adjacency matrix (that is, the eigenvalue of maximum modulus, and we note that this eigenvalue need not necessarily be unique). There are three cases according to whether the spectral radius is less than 1, greater than 1, or exactly 1:

- Case 1: Spectral radius < 1. A^n tends to zero as n tends to ∞, thus the ripples of the initial perturbation will eventually die out, leaving a stable non-zero total displacement.
- Case 2: Spectral radius > 1. The norm of A^n tends to ∞ as n tends to ∞, and so at least one element of A^n tends to ∞. Thus the initial perturbation has the potential to produce unstable ripples, also leaving an infinite total displacement (see discussion below).
- Case 3: Spectral radius = 1. The norm of A^n either tends to some constant value as n tends to ∞, or grows more slowly than geometrically (Gelfand's formula). Thus the effects may eventually stabilise to a constant value or may continue to grow slowly. In either case, the total displacement will not have a finite value. Note that even in the case where a constant perturbation exists, the total displacement cannot have a finite value as the perturbation never stops, and so the total displacement must grow without bound.

Though a spectral radius < 1 guarantees a finite total displacement, this does not rid us of the need to conduct the matrix multiplication described in the previous sections in those cases where the dominant eigenvalue is ≥ 1. The reason for this is that, depending on the nature of feedback cycles within the system and the precise nature of the perturbation, unstable behaviour may or may not permeate the system. This means that it is possible for some perturbations to produce stable consequences and others to produce unstable consequences in systems where the entire adjacency matrix has spectral radius ≥ 1.In all cases the matrix multiplication with the initial perturbation is necessary to determine the total change.

In this type of model, combining graphs generated by different stakeholders is more complicated than in FCM where degrees of certainty about causality can be improved by incorporating the knowledge of more experts and stakeholders additively (Kosko, 1988). There is debate about whether the weights should be averaged or some other method used to obtain a composite graph (Ozesmi and Ozesmi, 2004).

In summary, this interpretation of the graph is essentially a linear system dynamics model. It is not fuzzy since there is no uncertainty included in the model, just magnitude of effect. This interpretation completely changes the nature and implementation of the participatory process: where are one explores uncertain knowledge asking participants "how certain are you that A causes B", the other explores perceived relative magnitude of effect asking "what is the magnitude of the effect A has on B, how strong is the effect compared to other weights you have assigned".

While it is not fuzzy, building a simple systems dynamics model is very useful to do with participants in many cases, particularly when it is desirable to quantify the effect of different interventions on different variables. System dynamics models permit estimates such as "increasing fertilizer subsidies by 20% decreases deforestation by 5%", according to the worldview represented in the model. The process of building the system dynamics model allows for an exploration of different stakeholders' system boundary judgments and perceptions of the relationships elements of the system of interest as in the case of FCM.

Note aggregating graphs generated by different stakeholders is much more complicated in the case of system dynamics models as we are not simply becoming more certain, the more people say a particular link exists. Not surprisingly, studies using the system dynamics interpretation have shown that aggregating maps from different stakeholder groups can have very different outcomes about the perceived impacts under the same scenario given variation in knowledge structuring across groups, and the aggregation technique itself has a significant impact on outcomes (Gray et al., 2012).

6 What is the difference?

This section uses a single weighted directed graph to illustrate that there are significant consequences for decision-making if the same graph is interpreted as an FCM (Kosko, 1986) or as it has recently been appropriated in environmental decision-making, as a system dynamics model (Kok, 2009). The weighted graph in Figure 3 was chosen, as it is deliberately simple and abstract. Recall the adjacency matrix A is given in Equation 2.

Interpretation of graphs is not arbitrary; each interpretation of the weights on links and the state of the concepts corresponds to a specific mathematical formulation, which determines the meaning of the results in that context. While we can choose how to interpret the state of the concepts and the meaning of the weights on the links in formulating the graph, this choice specifies the mathematics and its meaning;

we cannot choose to apply arbitrary mathematics to arbitrary interpretation and the results are not then “up for re-interpretation”.

Within each interpretation there *is* a mathematically correct way to go about calculations. Various published FCMs in the environmental literature use equations that are inconsistent with the interpretation of the graph, or are simply internally mathematically incorrect, as discussed above. The following subsections apply mathematically correct calculations, consistent with each interpretation.

6.1.1 Interpretation as an FCM

Interpreting the graph in Figure 3 as an FCM, the weights are interpreted as a measure of how certain we are that one concept causes another to happen, or not to happen. The states are interpreted as a measure of how confident we are that a certain concept is activated (happening) or not.

It is possible to calculate what is “caused” given any initial conditions using Equation 6 for memoryless FCM or Equation 8 for one state memory. For the sake of this illustration we set the initial conditions as [1, 0, 0.5], and use the same vector in the system dynamics interpretation for comparison. We use the unipolar sigmoidal function given in Equation 4 as the squeezing function for the FCM with the parameter $\lambda = 0.5$.

The successive state activation calculated using Equation 6 for a memoryless FCM are shown in Figure 8. Quite quickly C1 settles at 0.54, C2 at 0.53 and C3 at 0.47. As all of these numbers are very close to 0.5 (perfect uncertainty about whether something is happening or not), we are very uncertain about the condition of C1, C2 and C3 given the initial conditions. What this means is that starting from certainty that C1 is happening, C2 is not happening and complete uncertainty about what C3 is doing, we cant really tell what will happen with the knowledge we have, though we lean towards C1 and C2 happening and C3 not happening, but not with much confidence. In fact, the fixed point (0.54,0.53,0.47) is reached from all starting conditions.

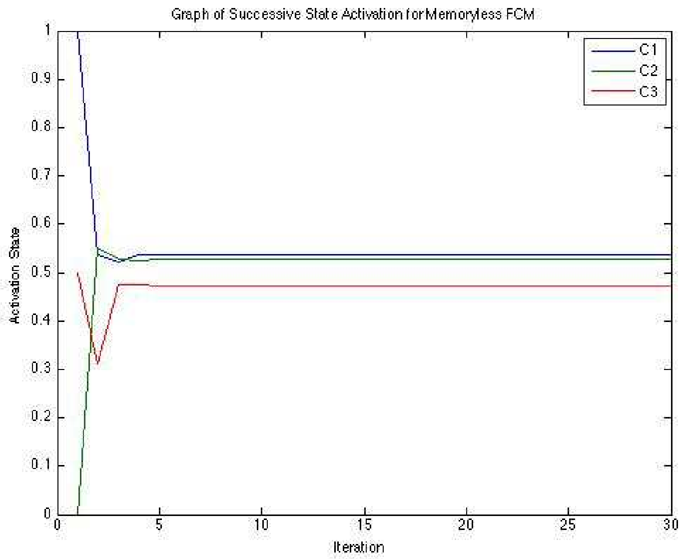

Figure 8 Graph of Successive State Activation calculated using Equation 6

The successive state activation calculated using Equation 8 for one state memory FCM are shown in Figure 9. Quite quickly C1 settles at 0.59, C2 at 0.59 and C3 at 0.55. As all of these numbers are very close to 0.5 (perfect uncertainty about whether something is happening or not), we are uncertain about the condition of C1, C2 and C3 given the initial conditions, although slightly less uncertain than in the memoryless case. As in the previous case, we cant really tell what will happen with the knowledge we have. We lean towards all concepts are happening but not with much confidence. In fact, the fixed point (0.59,0.59,0.55) is reached from all starting conditions for this FCM.

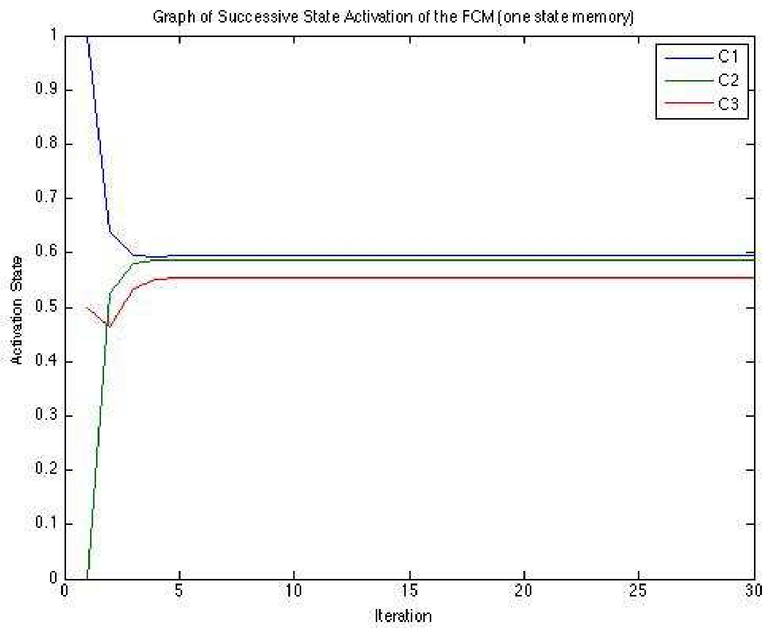

Figure 9 Graph of successive state activation calculated using Equation 8

What both of these FCM calculations have told us is how certain we are that certain things are happening or not happening based on a set of initial conditions. In this case we are quite uncertain about everything. Accordingly, this example also illustrates that in some cases the FCM serves to highlight how uncertain the inferences we make based on our knowledge systems are. Potentially this could be helpful in decision-aiding and open up space for entertaining alternative worldviews, more information, and humble and adaptive decision-making.

6.1.2 Interpretation as a system dynamics model

Interpreting the graph in Figure 3 as a system dynamics model, the weights are interpreted as measuring how a change in the value of one variable is transmitted to a change in the value of the connecting variable. In the simplest case of a linear system dynamics model, the weight is the slope of the straight line describing the relationship between the two concepts. This line could have any slope $\in [-\infty,\infty]$. There is no reason the weights on the links need to be $\in [-1,1]$ since a small change in one variable could produce a larger change in another variable. More general system dynamics models may include non-linear functions to describe the relationships between variables.

The state of each concept is its magnitude in real terms, it could be normalized to be $\in [0,1]$ or $[-1,1]$ or not. Functions with bounded support may be used to calculate the state values in cases where variables have finite range, however, pushing the sum of inputs through a squeezing function does not make sense in this interpretation. No squeezing function has been used here for mathematical correctness. The vector that multiplies the adjacency matrix, **A** is the perturbation, or change applied to the system. Note that in this type of system dynamics model nothing is changing unless an external push is applied.

Interpreting the graph in Figure 3 as a system dynamics model, it is possible to calculate the effect of an initial perturbation using Equation 11. For consistency the initial perturbation is set to [1, 0, 0.5]. In this model the initial perturbation eventually dies out so that in Figure 10 the successive changes become 0.

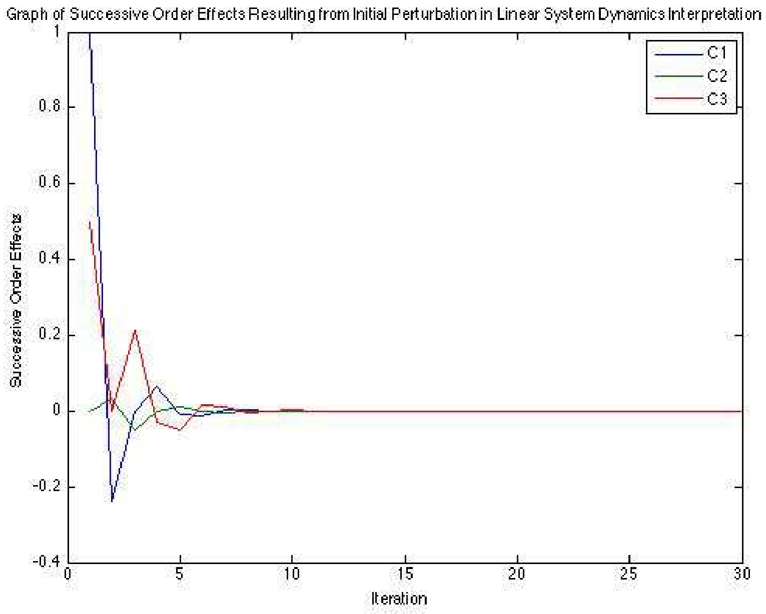

Figure 10 Graph of successive order effects resulting from initial perturbation in a system dynamics interpretation of the graph in Figure 3.

The cumulative change, calculated using Equation 13, resulting from the initial perturbation at each iteration is shown in Figure 11. The final change that resulted from the initial perturbation is C1 increased by 0.81, C2 decreased by 0.01 and C3 increased by 0.65.

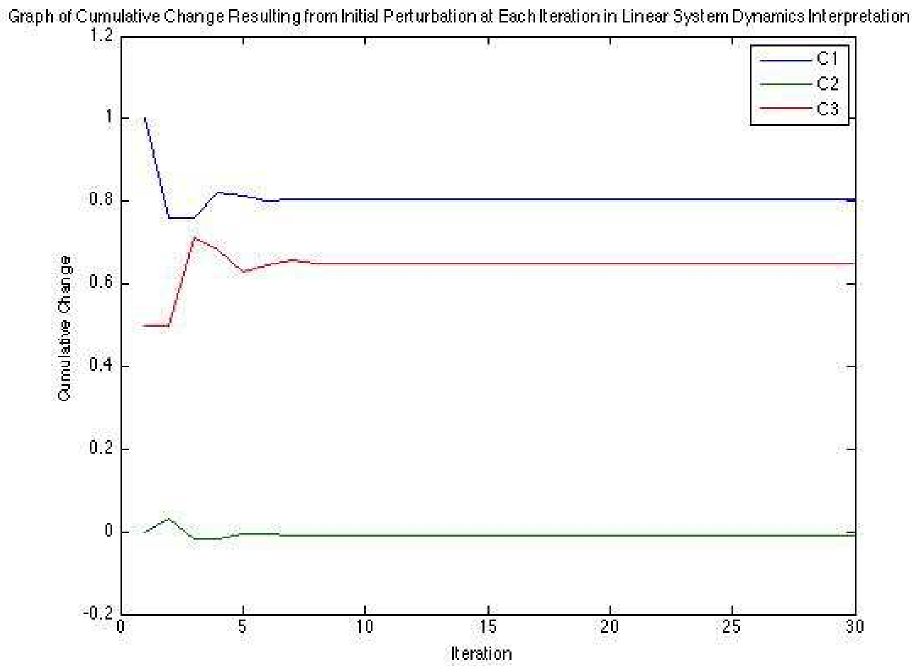

Figure 11 Cumulative change at each iteration, resulting from the initial perturbation in a system dynamics interpretation of the graph in Figure 3.

In this example the initial state of the system was not originally given. Say it is [0,1,0], then the final state of the system as a result of the perturbation would be [0.81, 0.99, 0.65]. The model does not explicitly tell us how long these changes take to occur. However, the estimation of relative weights does rely on a notion of a time unit, and thus each iteration could be thought of as one time unit.

There are situations where a system dynamics model might be desirable to have, as well as or instead of an FCM, and a linear first approximation could be a good start and provide some insights before introducing more complexity and non-linearity. In cases where linearity is obviously not a reasonable assumption, nonlinear system dynamics models can be built, the downside being that more expertise is often required for computation in this case.

6.1.3 Discussion

Taking exactly the same graph, one can obtain entirely different inferences for decision-making depending on how the graph is interpreted and calculations carried out. Accordingly, it is important that there is consistency in the interpretation that is used to generate the graph, to undertake calculations and to interpret the results.

The option to mix and match equations with interpretations according to fancy does not exist without producing mathematical gibberish. Equations are equivalent to sentences, describing the relationships between the components of the system and how they change relative to each other, and as such they have physical interpretations. The physical interpretation that gives rise to the equations, used consistently throughout calculation and interpretation of results, produces meaningful understanding.

FCM models can tell us what our knowledge system indicates will be caused from a particular intervention or any given set of initial conditions, together with the uncertainty associated with this inference. The system dynamics version can tell us the magnitude and direction of change of the values of the variables in the model, for a given initial perturbation. This form of the model does not include time it only models how the variables change with respect to each other not time. Accordingly, it does not tell us the time frame these changes take place in, and is not changing unless an external change is applied to one of the variables since 0•A=0.

The system dynamics interpretation of the weighted directed graph developed in participatory processes are not fuzzy at all, there is no uncertainty in the model[1]. The links that are there exist with complete certainty, the strength of the relationship between variables is also a fixed number and keeping it between 0 and 1 does nothing save limit the slope of the line between two variables and eliminate the

[1] Fuzziness could potentially be added by asking people for a range of values for the weight on the link and how certain they feel about this estimate.

possibility that a small change in one variable might produce a larger change in another.

For FCM the results are sensitive to choice and parameterization of squeezing function and to the amount of memory included. For system dynamics there is no reason to limit the weights or the state values to an absolute value between 0 and 1, and pushing the sum of the inputs to a concept node through a squeezing function is mathematically nonsensical.

7 Conclusion

"FCMs" are being used extensively to aid social and environmental decision-making processes, to capture different worldviews and explore the causes and effects of different variables. In the practice of social-environmental decision-aiding, it turns out that many FCMs are not fuzzy, and are in fact system dynamics models with some inherited mathematical features from the original formulation of FCMs[2] consistent with that interpretation physically and mathematically. This may be because in many cases system dynamics models and supply the information stakeholders and researchers are interested in: the magnitude and direction of change in the values of variables that results from particular disturbances or interventions.

FCMs, neural networks and system dynamics models can all be constructed through participatory processes. The process of defining the boundaries of the system of interest by deciding what factors are important to include and what can be ignored, how these factors relate to each other, which disturbances the system should be subjected to, and which interventions are possible, is hugely useful irrespective of the type of model being built or even whether a model is being built at all.

However beyond this FCM, neural networks and system dynamics models are different models that require different data in their formulation and answer different questions. FCMs have a structural relationship with neural networks and in the case where an unsupervised learning algorithm is applied to adjusting the weights of the FCM, the FCM is a neural network. However there is a fundamental difference between FCMs and system dynamics models in that one models uncertain and ambiguous causes and the other models unambiguous effects. For FCMs and system dynamics models the placing of weights on the links of the graph is very different. At this stage in the process FCM asks how certain are you that this causal link exists whereas system dynamics models ask what is the magnitude of effect of one variable on the other.

[2] For example, as limiting link weights to an absolute value less than 1, using a squashing function on the value of the state of each concept, or even putting 1's on the diagonal of so called "drivers". These conditions make physical and conceptual sense in the FCM formulation but not in the system dynamics model.

Though FCM and system dynamics models are somewhat confabulated in the social-ecological and resilience literature (Kok, 2009) it *is* important to differentiate between fuzzy cognitive maps and system dynamics models in this field. The choice of FCM versus system dynamics model changes:

- How the activity is framed and explained to participants in terms of what is being done, what the purpose is and what outcomes and outputs should ideally be.
- It changes what questions you should ask participants and what data to collect. For FCM links, "how certain are you about the existence of this causal relationship" versus, "how much does this variable affect the other" are very different questions which are likely to produce different weighted graphs.
- It changes the type of information you can get from the model. FCM models can tell us what our knowledge system indicates will be caused from a particular intervention or any given set of initial conditions, together with the uncertainty associated with this inference. The system dynamics version can tell us the magnitude and direction of change of the values of the variables in the model, for a given initial perturbation.
- It changes the mathematical rules of the calculations that need to be applied.

All of these items have a significant impact on the decision-aiding process and the conclusions and recommendations for action drawn. A coherent mathematical model with consistent conceptual interpretation and practical implementation is important for the quality of the results and the meaning derived from the exercise. The clearer practitioners are themselves about what is going on here, the more simply they can explain the method and more readily they can facilitate the understanding of participants, collect meaningful representations and conduct logically rigorous and meaningful calculations providing useful results, robust conclusions and recommendations.

Some might argue that considerations relating to mathematical calculations don't matter, because the important thing is having a legitimate reason to get participants together to discuss their views and it is the discussion and the map they draw that matters. Indeed, "the process of creating a model with stakeholders is often worthwhile in and of itself for its engagement value, and for the insights offered to the researcher" (Knight et al., 2014). However, further calculations *are* done, and that this is the case is often part of legitimating the activity and drawing in the stakeholders. Also the faith people have in a methodology that is perceived to be advanced mathematically "might cause results to be accepted without a prudential degree of skepticism"(Kontogianni et al., 2012). Since the inferences made as a result of the activity are highly sensitive to the considerations above, it is important that those calculations are meaningfully conducted and interpreted.

The cognitive maps (elements and linkages) generated by participants are used in further analytic techniques, and through the analysis can crucially shape the mindsets of participants and assessment of the merit of any proposed policies and programs in the ultimate decision-making process (Ulrich, 1983). It is therefore equally key to ethical and effective policy and program development that the analysis with its underlying mathematics be an accurate model and a correct inference of the knowledge elicited from participants.

FCMs continue to be used and continue to have a lot to offer, so do system dynamics models, though they serve different purposes. Work has been done to put cause and effect together. Theoretically it should be possible to ask people where they think causal links exist, how certain they are that the causal link is there, and also what the magnitude of effect is. In order to 'fuzzify' the magnitude of effect, participants could be asked for a range of values instead of just one. This is potentially easier and quicker than estimating single values as the entire range suggested by all participants can be adopted. Work is currently being done on mathematical approaches which incorporate uncertainty in both cause and effect in such ways (Papageorgiou and Salmeron, 2013). FCMs, system dynamics and other forms of participatory modeling contain many possibilities for further research and development. Harnessing technology to involve as much of the population as possible in fun or gamelike model development is another interesting topic.

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